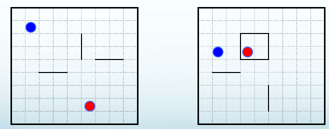
The Combinatorics of Quoridor® Game States

Ryan Campbell
Monroe Griffin

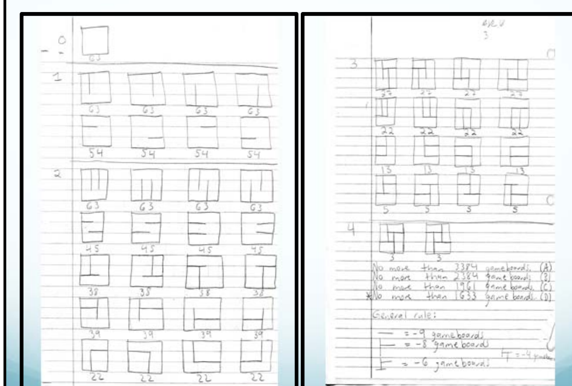
Game State

- A possible position that players can occupy legally
- No matter how crazy
- Determines complexity of game
  - The overall total number



3x3

- Total Games States = 1633
- 5x5….
- Lead to general formula



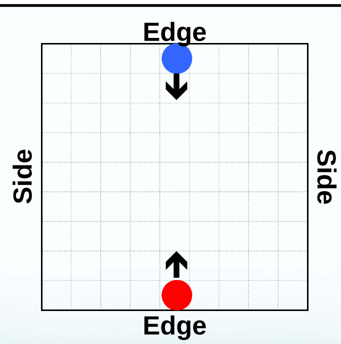
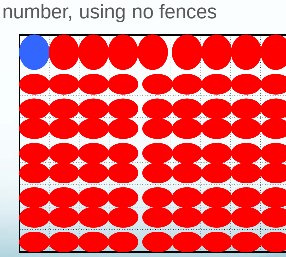


Diagram of Quoridor Board

Total Number of Game States

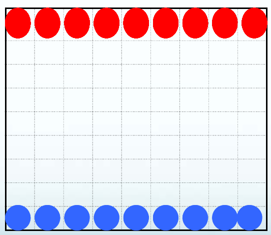
- Each Game Board has a cap
- The total number, using no fences
- To find:



Total = Area x (Area -1)

Adjusted Total

- Both players cannot win at the same time

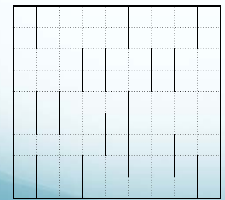


- Subtract Area
- **So far, Total (cap) = Area x (Area -1) - Area**

Types of Fences

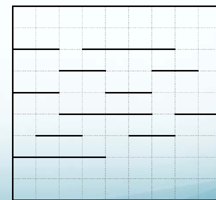
Vertical

- No effect

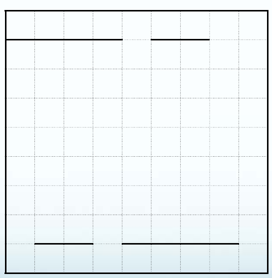


Horizontal

- Not against edge
- No effect

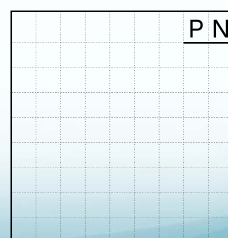


Horizontal Fences Against Edge

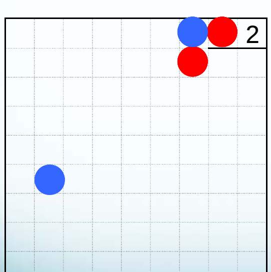


Our Formula

- Works for individual game boards
- Number of Game States to Subtract from adjusted total
- P = Possible
- N = Not Possible

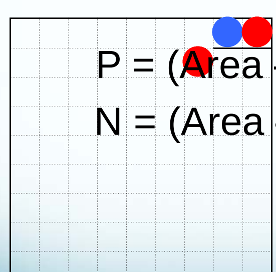


Possible Vs. Not Possible



P = (Area – 2) – side

Possible Vs. Not Possible



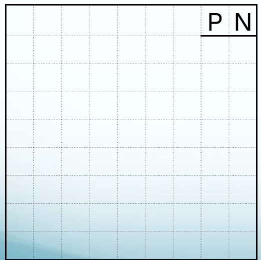
P = (Area – 2) – side

N = (Area – 1) – side

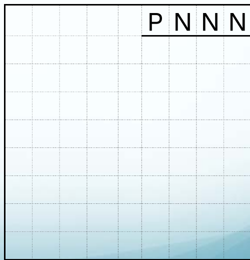
N = (Area – 1) – side

Fences Touching Side

One Fence



Two Fences

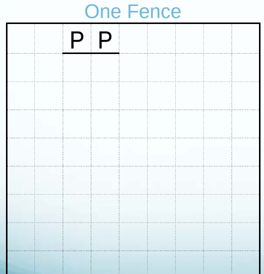


Fences Touching

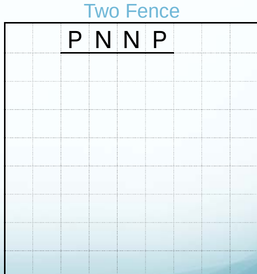
Fences	Formula
1	N + P
2	3N + P
3	5N + P
4	7N + P
(2F -1) x N + P	

Fences Not Touching Sides

One Fence



Two Fence



Fences Touching

Fences	Formula
1	N + P
2	3N + P
3	5N + P
4	7N + P
(2F -1) x N + P	

Fences Not Touching

Fences	Formula
1	2P
2	2N + 2P
3	4N + 2P
(2F -2) x N + 2P	

Difference of One

- Fences Touching
- Fences Not Touching

Fences Touching	Fences Not Touching
(2F -1) x N + P	(2F -2) x N + 2P
2F(N) – N + P	2F(N) – 2N +2P
2F(A – 1 – s) – (A – 1 – s) + (A– 2 – s)	2F(A – 1 – s) – 2(A – 1 – s) + 2(A– 2 – s)
2F(A – 1 – s) – ~~A~~ + 1 + ~~s~~ + ~~A~~ – 2 – ~~s~~	2F(A – 1 – s) – ~~2A~~ + 2 + ~~2s~~ + ~~2A~~ – 4 – ~~2s~~
2F(A – 1 – s) – 1	2F(A – 1 – s) – 2

Our Formula

$2F(s^2 - s - 1) - C$, where $C = \begin{cases} 1 & \text{if touching} \\ 2 & \text{if not touching} \end{cases}$

$2F(81 - 9 - 1) - C$

$2F(71) - C$

$142F - C$

What's Next

- Other fence configurations
  - Boxes
  - More complex arrangements
- Total number of game states for 9x9

Questions?

Thank you!