The Combinatorics of Quoridor® Game States

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Game State
- A possible position that players can occupy legally
- No matter how crazy
- Determines complexity of game
  - The overall total number

Possible
Not Possible

Total Games States = 1633
5x5...
Lead to general formula

3x3
- Total Games States = 1633
- 5x5...
- Lead to general formula

Diagram of Quoridor Board

Total Number of Game States
- Each Game Board has a cap
- The total number, using no fences
- To find:

Total = Area x (Area - 1)
Adjusted Total

- Both players cannot win at the same time
- Subtract Area
  
  So far, Total (cap) = Area x (Area -1) - Area

Types of Fences

- Vertical
  - No effect
- Horizontal
  - Not against edge
  - No effect

Horizontal Fences Against Edge

Our Formula

- Works for individual game boards
- Number of Game States to Subtract from adjusted total
- P = Possible
- N = Not Possible

Possible Vs. Not Possible

Possible Vs. Not Possible

\[
P = (\text{Area} - 2) - \text{side}
\]

\[
N = (\text{Area} - 1) - \text{side}
\]
**Fences Touching Side**

- One Fence
  - P N

- Two Fences
  - P N N N

**Fences Not Touching Sides**

- One Fence
  - P P

- Two Fence
  - P N N P

**Fences Touching**

<table>
<thead>
<tr>
<th>Fences</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N + P</td>
</tr>
<tr>
<td>2</td>
<td>3N + P</td>
</tr>
<tr>
<td>3</td>
<td>5N + P</td>
</tr>
<tr>
<td>4</td>
<td>7N + P</td>
</tr>
<tr>
<td></td>
<td>(2F - 1) x N + P</td>
</tr>
</tbody>
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**Fences Not Touching**

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**Difference of One**

- Fences Touching
  - \((2F - 1) x N + P\)
  - \(2F(N) - N + P\)
  - \(2F(A - 1 - s) - (A - 1 - s) + (A - 2 - s)\)

- Fences Not Touching
  - \((2F - 2) x N + 2P\)
  - \(2F(N) - 2N + 2P\)
  - \(2F(A - 1 - s) - 2(A - 1 - s) + 2(A - 2 - s)\)

**Our Formula**

\[2F(s^2 - s - 1) - C, \text{ where } C = \begin{cases} 1 & \text{if touching} \\ 2 & \text{if not touching} \end{cases}\]

- \(2F(61 - 9 - 1) - C\)
- \(2F(71) - C\)
- \(142F - C\)
What's Next

- Other fence configurations
- Boxes
- More complex arrangements
- Total number of game states for 9x9

Questions?

Thank you!